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Hong Kong - Singapore joint Seminar Series in Financial Mathematics/Engineering

GANs as Gradient Flows that Converge

Prof. Yu-Jui Huang
University of Colorado, Boulder

Abstract

This paper approaches the unsupervised learning problem by gradient descent in the space of probability density functions. Our main result shows that along the gradient flow induced by a distribution-dependent ordinary differential equation (ODE), the unknown data distribution emerges as the long-time limit of this flow of densities. That is, one can uncover the data distribution by simulating the distribution-dependent ODE. Intriguingly, we find that the simulation of the ODE is equivalent to the training of generative adversarial networks (GANs). The GAN framework between a generator and a discriminator can therefore be viewed alternatively as a cooperative game between a navigator and a calibrator. At the theoretic level, this new perspective simplifies the analysis of GANs and gives new insight into their performance. To construct a solution to the distribution-dependent ODE, we first show that the associated nonlinear Fokker-Planck equation has a unique weak solution, using the Crandall-Liggett theorem in Banach spaces. From this solution to the Fokker-Planck equation, we construct a unique solution to the ODE, relying on Trevisan's superposition principle. The convergence of the induced gradient flow to the data distribution is obtained by analyzing the Fokker-Planck equation. This is joint work with Yuchong Zhang.

About the speaker

Prof. Yu-Jui Huang is an Assistant Professor in Department of Applied Mathematics at the University of Colorado Boulder since 2016. He obtained his Ph.D. degree at University of Michigan and worked as a Lecturer at Dublin City University. His research is focused on mathematical finance, time inconsistent stopping and control, mathematical foundations of machine learning. He has received SIAM SIGEST Award and 2015 Bruti-Liberati Fellow.

Date

July 21, 2022 (Thursday)
(HK Time)

Time

11:15 am – 12:15 pm
(HK Time)

Zoom

https://polyu.zoom.us/j/96735361463?pwd=bkFWK09uNzBNU2RjVm9KVlFPTy8zQT09

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